Time Domain Characterization of Lossy Arbitrary Characteristic Impedance Transmission Lines

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***Abstract*—This paper deals with the characterization of lossy transmission lines. The method developed here delivers the complex propagation constant γ of any arbitrary length and characteristic impedance transmission line, embedded in an arbitrary environment. This approach is based upon time domain analysis of short pulse propagation. Measurements are done with a commercial digital sampling oscilloscope. Only two transmission lines of different lengths are required in order to extract γ and to correct systematic errors of the measurement system. The problem of random errors is also addressed. The method is demonstrated with microstrip lines. A comparison of the developed technique with an other existing time domain approach and a classical frequency domain extraction is also carried out.**

I. Introduction

It is now well known that interconnects are a bottleneck in the development of high-performance circuits. Therefore, it is often necessary to fully characterize the transmission lines that carry the fastest signals in the circuit. For signals of about 150 ps risetime, it is accepted that the characterization of interconnection lines have to be done over the whole bandwidth from dc to 20 GHz.

Transmission lines can be characterized from measurements obtained either in frequency domain (FD) or in time domain (TD). In FD, measurements are generally done with a Vector Network Analyser (VNA). Such a system must be calibrated [1]. This procedure requires a full set of calibration standards. The method proposed here is rigorous, but it is much more simple. Measurements are done in TD by a commercial Digital Sampling Oscilloscope.

The TD approach has already been used in [2], [3] for the characterization of resistive lines. The method described in [2] only requires the measurement of one line, but the applicability of the method is limited: the setup and the line under test must be well-matched to 50 Ω in order to avoid multiple reflections effects that cannot be taken into account and corrected. The method has been applied to a 3-m-long 50-Ω coaxial Teflon cable. This example does not reveal the limitation of the method since internal reflections are negligible in this configuration. In the case of short mismatched transmission lines, these effects are dramatic. The method described in [3] allows to deal with mismatched lines. It is based on the measurement of two identical lines of different lengths. The ratio of the Fourier Transform of the two related forward travelling waves directly gives the complex propagation coefficient γ of the line under characterization. A convenient time windowing allows to avoid parasitic reflections. Nevertheless, this method suffers from two drawbacks. First, the TDR-TDT system time shifts cannot be corrected because only transmitted signals are measured. This gives significant errors (see Section IV). Secondly, the rejection of unwanted reflections (for the characterization of mismatched lines) must be made by windowing. This leads to practical limitations with commercial oscilloscopes (TekCSA803 or HP54120): the electrical length of the shortest line to be measured must be greater than about 250 ps because generators reach their steady-state very slowly, after multiple ripples (see Section IV). For high-loss lines, this constraint is even more severe because the steady state of the forward-travelling waves is reached even slower. That makes this method unsuitable for MMIC lines characterization with oscilloscopes.

In this paper, like in [3], only two identical lines of different lengths are required. Conversely, in order to correct measurement errors for any arbitrary length and characteristic impedance lines, bi-directional measurements are done. γ is then extracted from these measurements by a de-embedding procedure.

Section II describes the theoretical development of the method. The problem of measurement errors is addressed in Section III and experimental results are given in Section IV.

II. Theoretical Development

Engen [4] has shown that the complex propagation constant of an unknown transmission line can be calculated from S-parameters of two sections L1 and L2 of unequal length of this line. The complex propagation coefficient γ is given by:

$$\exp(\gamma\Delta 1) = \frac{t_{11} + t_{22} \pm R}{t_{11} + t_{22} \mp R} \quad (1)$$

with:

$$R = \sqrt{(t_{11} - t_{22})^2 + 4t_{21}t_{12}}, \quad (2)$$

$$T = R_{L2}R_{L1}^{-1} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}, \quad (3)$$

Δl: length difference between the two sections,

R_{L1}, R_{L2}: Wave Cascading Matrix corresponding to the S-parameters of sections L1 and L2,

$$\gamma(\omega) = \alpha(\omega) + \beta(\omega).$$

$\alpha(\omega)$, and $\beta(\omega)$ are the frequency dependent attenuation coefficient and the propagation constant, respectively.

Manuscript received January 26, 1994.

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IEEE Log Number 9402180.

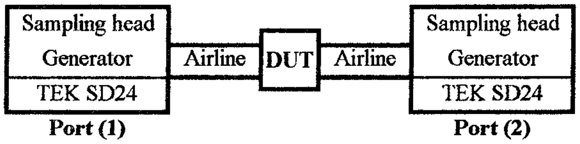

Fig.1. Bi-directional TDR-TDT system.

Equation (2) can also be written:

$$R = \sqrt{(t_{11} + t_{22})^2 + 4(t_{21}t_{12} - t_{11}t_{22})}. \quad (4)$$

Then, by writing $A = t_{11} + t_{22}$ and $B = t_{11}t_{22} - t_{12}t_{21}$ and, it comes :

$$\gamma = \frac{1}{\Delta l} \ln \left(\frac{A \pm \sqrt{A^2 - 4B}}{2} \right). \quad (5)$$

A and B can also be expressed as a function of the S-parameters of the sections $L1$ and $L2$ (see (6) and (7), at the bottom of this page) with:

$$\Delta_k = S11_k S22_k - S12_k S21_k \qquad k = L1 \text{ or } L2. \quad (8)$$

The TDR-TDT bi-directional system is shown in Fig. 1. The Device UnderTest (DUT) can be excited either in forward or backward direction. The impedance of each generator (Step Recovery Diodes) is different between "ON" and "OFF" states. So, the mismatch of port i (i = 1 or 2) due to the intrinsic measurement system is different whether the generator of port i is "ON" or "OFF". To overcome this problem, the simple solution of time windowing can be used. Two good quality lines are connected between the two measurement ports and the DUT, as shown in Fig. 1. The return loss along these windowing lines must be less than -25 dB, in order to provide a quasi-constant impedance and no significant internal reflections. One should note that even without windowing, these lines are always necessary to separate forward and backward travelling waves. For comparison, in superheterodyne VNA, this is achieved by the couplers.

After windowing, forward- and backward- travelling waves of $L1$ and $L2$ are measured. The recorded waveforms are noted $bijk(t)$; i is relative to the excitation port (generator 1 or 2), j to the measurement port (sampling-head 1 or 2) and k to the section to be measured ($L1$ or $L2$). Then, the Complete Fast Fourier Transform [5] is applied to $bijk(t)$ waveforms, leading to $Bijk(\omega)$ parameters. These $Bijk(\omega)$ parameters can be related to the corresponding $Sijk(\omega)$ parameters in (9), shown at the bottom of this page, with $Gi(\omega)$ the frequency response of the generators Ports (1) and (2), and $Ei(\omega)$ the frequency response of the sampling heads ports (1) and (2).

By substituting the $Sijk(\omega)$ parameters deduced from (9) in $A(\omega)$ and $B(\omega)$ (see (6) and (7)), it can be easily shown that $A(\omega)$ and $B(\omega)$ are independent of the $Ei(\omega)$ and $Gi(\omega)$. So, the knowledge of the $Bijk(\omega)$ parameters is sufficient in order to extract γ from (5).

III. Measurement Errors

For actual measurement systems, errors can be separated into systematic and random errors. A complete description of systematic errors is given in [6]. These errors are intrinsically completely corrected by using the method described in Section II.

For our system (TekCSA803), the random errors sources are noise, jitter, and generators time shifts.

- Noise is due to oscilloscope amplifiers and generators (diodes); its standard deviation is about 0.6 mV.
- Jitter is due to an imperfect trigger and can be modelled as a timebase noise. Its standard deviation is 3.5 ps.
- Generators time shifts are due to thermal effects on the triggering of the diodes. It can reach a maximum value of 1ps/mn for temperature deviations of less than 0.05°/mn [7]. Random errors can be divided in two categories, depending on whether they can be corrected or not by the use of models. Generators time shifts can be modelled as pure delays [7]. So, these drifts correspond to a phase difference in FD. Then, drifts occurred between measurements of $L1$ and $L2$ are corrected in FD. The chosen phase reference is that of the incident generator waveform of the first section measured by TDR for the two ports.

The other errors (noise and jitter) cannot be completely corrected, even by the use of statistical models, because the generator time shifts are not linear with the time. However, they are minimized by an optimal averaging [7] that corresponds to a compromise between the decrease of random errors effects and increase of the system nonstationarity effects.

IV. Experimental Results

The complex propagation coefficient of a microstrip line has been determined. Sections $L1$ and $L2$ are 14 and 49 mm long, respectively. These lines are fabricated on a 635-μm-thick 99.6% alumina substrate. Conductors are made with ESL5835 conductive paste; they are 220 μm wide and 10 μm thick.

$$A = \frac{(S11_{L2} - S11_{L1})(S22_{L1} - S22_{L2}) + S12_{L1}S21_{L1} + S12_{L2}S21_{L2}}{S21_{L2}S12_{L1}}. \quad (6)$$

and

$$B = \frac{(S11_{L2}S22_{L1} - \Delta_{L2})(S11_{L1}S22_{L2} - \Delta_{L1}) - (\Delta_{L2}S11_{L1} - \Delta_{L1}S11_{L2})(S22_{L1} - S22_{L2})}{(S21_{L2}S12_{L1})^2}. \quad (7)$$

$$\begin{aligned} B11k(\omega) &= S11k(\omega)G1(\omega)E1(\omega)\ ; \quad B21k(\omega) = S21k(\omega)G1(\omega)E2(\omega); \\ B12k(\omega) &= S12k(\omega)G2(\omega)E1(\omega)\ ; \quad B22k(\omega) = S22k(\omega)G2(\omega)E2(\omega); \end{aligned} \quad (9)$$

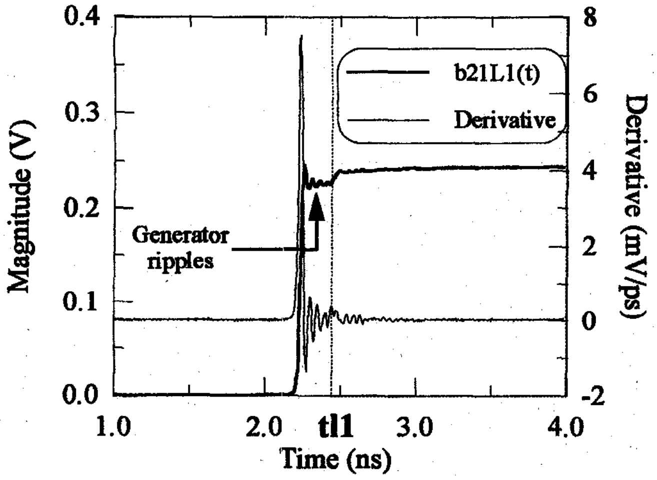

Fig. 2. Line L1 forward-travelling wave measurement and its derivative.

A quasi-static computation gives a characteristic impedance equal to 78 Ω.

Fig. 2 shows the forward-travelling wave measured on the shortest section $L1$, and its derivative. Multiple ripples of the generators have to be noted. Fig. 3 shows the results obtained for α and $\beta n = \beta(\omega)/\beta_0$ (with $\beta_0^2 = \omega^2\varepsilon_0\mu_0$) by our method and for comparison, by the method described by Deutch in [3]. The same TD measured waveforms have been used with the two approaches. For the method described in [3], time windows have been shortened to $tl1$ (see Fig. 2) for $b21_{L1}(t)$, and to $tl2$ for $b21_{L2}(t)$, in order to reject unwanted reflections. Next, the Complete Fast Fourier Transform has been used to avoid aliasing errors when computing the Fourier Transform. The derivative of $b21_{L1}(t)$ shows that internal reflections have occurred before the directly forward-travelling wave has reached its steady state. This leads to significant errors on α and βn, as shown in Fig. 3. Moreover, the system time shift has not been corrected, leading to additional errors, mainly on the value of βn.

The experimental validation of our method is done by comparing our results to those obtained by a FD approach after a TRL calibration on a superheterodyne VNA. As it can be seen in Fig. 3, the agreement is very good. The difference between the two approaches is less than 8% on α and less than 0.5% on βn on the dc 20-GHz bandwidth.

V. Conclusion

It has been shown that it is possible to fully characterize transmission lines of arbitrary length and characteristic impedance by a TD measurement approach. The method developed requires only two sections of unequal lengths of an unknown line and corrects all the TD system systematic errors. This simple and rigorous method is very attractive for printed circuit designers who often work with commercial TDR-TDT systems.

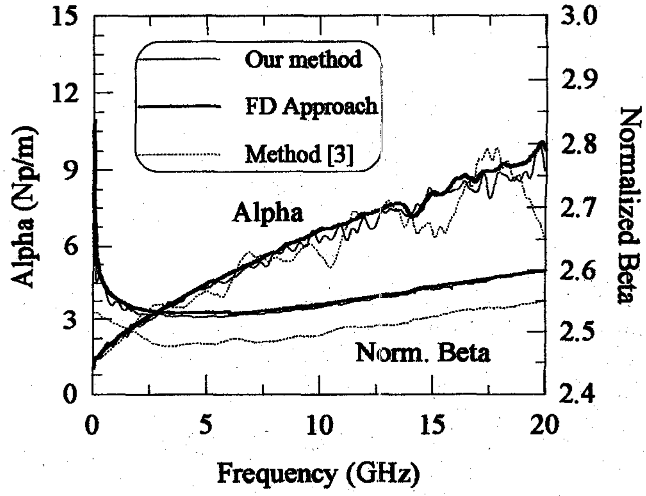

Fig. 3. Experimental results for α and βn obtained by TD approaches (our original method and that described in [3]) and a classical FD approach.

In a more general way, this work demonstrates the great capabilities of direct TD measurements in very large band applications. These capabilities will be still further increased with the development of very fast generators and samplers working in the picosecond or subpicosecond domain [9].

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